# Statistics of OBA stars magnetic fields 

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#### Abstract

Recent measurements of the OBA stars magnetic fields show their magnetic fields to be distributed according to the logarithm-normal law with an average value of $\log (\mathcal{B}) \sim 2.6-2.8$ and a standard deviation of $\sigma \sim 0.25-0.66$, where $\mathcal{B}$ is the root-mean-square magnetic field in G .

The fraction of OB stars with measured magnetic fields from $\sim 100 \mathrm{G}$ to $\sim 50 \mathrm{kG}$ (magnetic OB stars) is $7-12 \%$. Until recently, no magnetic fields were found in other OBA stars. Improvement of the technique for measuring magnetic fields over the past 5-7 years has made it possible to detect weak magnetic fields in $\sim 10$ BA stars with a B value in the range from 0.2 to $\sim 15 \mathrm{G}$. The distribution of the magnetic fields of these weakly magnetic stars can also be described by the log-normal law with the same variance as for magnetic OB stars but with an average $\log (\mathcal{B}) \sim 0.10$. By analogy with magnetic OBA stars, we assumed that this distribution is valid for all OBA stars with unmeasured magnetic fields.

With the radii of these stars we obtain the distribution of magnetic fluxes and magnetic moments of both magnetic and weakly magnetic stars. It is shown that these values for all magnetic OBA stars can be described by common distribution functions. The average magnetic fluxes $\Phi$ (in $G \cdot \mathrm{~cm}^{2}$ ) of magnetic and weakly magnetic OB stars are $\log (\Phi) \sim 26.3$ and $\sim 23.1$, which is close to the corresponding values for magnetars and radio pulsars, respectively. The data obtained allow us to conclude that there is a common distribution for the magnetic fields of all OB stars, both magnetic and weakly magnetic. Such distribution in dependence of a fraction of magnetic stars is presented. The magnetic moments of radio pulsars and magnetars are shown to be 5 orders of magnitude smaller than the corresponding values for weakly magnetic and magnetic stars.


Key words: stars, magnetic fields, statistics

## 1 Introduction

The origin of the magnetic fields of OBA stars remains enigmatic. The hypothesis that the stellar magnetic field may be relict was first suggested by Cowling (1945) who showed that the ohmic dissipation time of the magnetic field in stars with masses $M>1.5 M_{\odot}$ exceeds their lifetime and concluded that stellar magnetic fields may be a relic of the magnetic field of protostellar clouds. The idea of the relict nature of the magnetic fields of early spectral type stars was also argued by Moss (2003).

The numerical modeling of Braithwaite and Spruit (2004), Braithwaite and Nordlund (2006), Duez et al. (2010), and Duez and Mathis (2010) showed that there exist stable during the full stellar lifetime field configurations for various initial magnetic field distributions. Meanwhile, the fraction of magnetic OBA stars ${ }^{1}$ among all stars of these spectral types is only 7-12 \% according to, for example, Alecian et al. (2019).

[^0]The relatively small fraction of magnetic stars among all early spectral type stars has no clear explanation. One of the causes of this effect may be the hypothesis of Ferrario et al. (2009) that the magnetic fields of early-type stars are formed during the merging of protostars through a fast dynamo process.

New techniques for measuring magnetic fields and improving detectors over the past 5-10 years made it possible to detect weak magnetic fields in about 10 BAF stars with longitudinal magnetic fields in the range from 0.2 to $\sim 15 \mathrm{G}$ (Alecian et al., 2016; Blazère et al., 2016a, b). Thus, an array of early spectral type stars can be presented as a sum of a relatively small group of magnetic stars and the bulk of stars with low magnetic fields or weakly magnetic (WM) stars.

Kholtygin and Makarenko (2019) noted the similarity of two groups of OBA stars in relation to the magnitude of their magnetic fluxes and two groups of neutron stars normal neutron stars (radio pulsars) and magnetars - and suggested that radio pulsars are the descendants of WM stars, and magnetars are the scions of a group of magnetic stars. Makarenko et al. (2021b) showed that such a similarity is not exact.

It should be noted that the fraction of magnetars in the total bulk of neutron stars is only $1 \%$ compared to the fraction
$\sim 99 \%$ of normal neutron stars, which requires an explanation. A statistical study of the magnetic fields of OBA stars and a comparison of the distributions of their magnetic fields, magnetic fluxes, and magnetic moments with those obtained from an analysis of the magnetic properties of their descendants - neutron stars - can shed light on this problem. This problem is considered in the present paper.

The paper is organized as follows. Section 2 discusses the distribution of magnetic fields and magnetic fluxes of OBA and neutron stars, and their approximations. The distribution of the magnetic moments of these groups of stars is analyzed in Section 3. Some conclusions are given in Section 4.

## 2 Distribution of magnetic fields and magnetic fluxes

### 2.1 Statistical characteristics of magnetic fields

As a result of polarization observations of stars the longitudinal magnetic field $B_{z}$, also called the effective magnetic field, can be measured. The value of $B_{z}$ strongly depends on the rotation phase of a star and is not suitable for statistical studies of the magnetic fields for large ensembles of stars. For this reason, it is necessary to use such a global characteristic of the field which can be obtained from observations and at the same time weakly depends on the rotation phases at which the field measurements were made.

Determined by Borra et al. (1983) rms magnetic field is used as the most suitable characteristic of the stellar magnetic field weakly dependent on moments when the observations were made:

$$
\begin{equation*}
\mathcal{B}=\sqrt{\frac{1}{n} \sum_{j=1}^{n}\left(B_{z}^{j}\right)^{2}} \tag{1}
\end{equation*}
$$

where the squares of all measured values of the mean longitudinal magnetic fields $B_{z}^{j}$ for a given star are summed. Here $j$ is the observation number, and $n$ is their total number. Kholtygin et al. (2010b) showed that for the dipole configuration of the magnetic field the value of $\mathcal{B}$ weakly depends on the phase of rotation of a star $\phi$, the inclination angle $i$ of the rotation axis, and the angle $\beta$ between the axis of rotation and the axis of the magnetic dipole. This conclusion is also valid for quadrupole and other magnetic field configurations.

Borra et al. (1983) defines the following parameters that determine the accuracy of magnetic field measurements:

$$
\begin{equation*}
\Sigma_{\mathcal{B}}=\sqrt{\frac{1}{n} \sum_{j=1}^{n} \sigma_{j}^{2}}, \tag{2}
\end{equation*}
$$

where $\sigma_{j}$ is the $r m s$ error of the $j$-th measurement of the field, and

$$
\begin{equation*}
\chi^{2} / n=\sqrt{\frac{1}{n} \sum_{j=1}^{n}\left(\frac{B_{z}^{j}}{\sigma_{j}}\right)^{2}} . \tag{3}
\end{equation*}
$$

The larger the ratio $\mathcal{B} / \sigma_{\mathcal{B}}$ and the value of $\chi^{2} / n$, the more realistic the magnetic field measurements are. To confirm the reality of measurements, one can use the criterion proposed by Kholtygin et al. (2011) (their formula (4)). For $n>3$, this criterion means that the absolute value of the measured magnetic field $\left|B_{z}\right|$ is 3 times larger than the error of measurement at least for one measurement.

### 2.2 Sources of magnetic fields and radii of stars

The measurements of OBA stars magnetic fields are taken from the papers of Alecian et al. (2014), Aurière et al. (2007), Briquet et al. (2007), Castro et al. (2015), Elkin et al. (2010), Folsom et al. (2013), Grunhut et al. (2009), Grunhut et al. (2012), Grunhut et al. (2013), Henrichs et al. (2012), Hubrig et al. (2008), Hubrig et al. (2012b), Hubrig et al. (2012a), Hubrig et al. (2014), Mathys (2017), Järvinen et al. (2017), Landstreet et al. (2008), Neiner et al. (2015), Shultz and Wade (2017), Romanyuk et al. (2017), Sikora et al. (2016), Sikora et al. (2019), Stütz et al. (2003), Wade et al. (2011), Wade et al. (2012a), Wade et al. (2012b), Wade et al. (2012c), Wade et al. (2015).

The data from the catalog of Bychkov et al. (2009) are also used. Magnetic fields for weakly magnetic stars are extracted from the papers of Alecian et al. (2016), Blazère et al. (2016a), Blazère et al. (2016b), Lignières et al. (2009), Neiner et al. (2017), Petit et al. (2011), Petit et al. (2013), Seach et al. (2020).

Stellar radii are taken from Aurière et al. (2007), Castro et al. (2015), Neiner et al. (2017), Rhee et al. (2007), Pasinetti Fracassini et al. (2001), Shulyak et al. (2014), Wade et al. (2012a), Wade et al. (2015).

The parameters of normal neutron stars (radio pulsars) are extracted from the catalog of Manchester et al. (2005), whereas the characteristics of magnetars are found from the catalog of Olausen and Kaspi (2014). We excluded from consideration millisecond pulsars, the nature and ages of which differ significantly from those for radio pulsars at the stage of evolution up to the dead line. For all neutron stars, both normal pulsars and magnetars, the average radius was taken as $R=10 \mathrm{~km}$.

The standard formula of Manchester et al. (2005) is used to determine the surface magnetic fields of radio pulsars and magnetars:

$$
\begin{equation*}
B_{\mathrm{s}}=3.2 \cdot 10^{19} \sqrt{P \dot{P}}, \tag{4}
\end{equation*}
$$

where $P$ is the rotation period for a pulsar or a magnetar and $\dot{P}$ is the rotation period derivative.

### 2.3 Distribution of OBA stars magnetic fields

The differential distribution of the magnetic field can be obtained using the following relation:

$$
\begin{equation*}
f(\mathcal{B}) \approx \frac{N(\mathcal{B}, \mathcal{B}+\Delta \mathcal{B})}{N \cdot \Delta \mathcal{B}} \tag{5}
\end{equation*}
$$

where $N(\mathcal{B}, \mathcal{B}+\Delta \mathcal{B})$ is the number of the $r m s$ magnetic field in the interval $(\mathcal{B}, \mathcal{B}+\Delta \mathcal{B}), N$ is the full number of stars with the measured magnetic field.


Fig. 1. Distributions of the $r m s$ magnetic fields for OBA and weakly magnetic stars.

The distributions $f(\mathcal{B})$ determined by us for OBA stars and for a group of weakly magnetic stars with measured magnetic fields are presented in Fig. 1 and can be described by the logarithm-normal law. Denote $\eta=\log (\mathcal{B})$, then

$$
\begin{equation*}
f(\eta)=f(\log \mathcal{B})=\frac{1}{\sigma_{\eta} \sqrt{2 \pi}} e^{-\frac{1}{2}\left(\frac{\eta-\bar{\eta}}{\sigma_{\eta}}\right)^{2}} \tag{6}
\end{equation*}
$$

where $\sigma_{\eta}=\sigma_{\log \mathcal{B}}$ is the standard deviation of the random value $\eta$.

When approximating the real distribution of magnetic fields by the logarithm-normal law one should take into account that the value of the rms magnetic field $\mathcal{B}^{\prime}$ obtained from the analysis of observations of a star may differ from the value of $\overline{\mathcal{B}}$. In the first approximation the errors in determining the rms magnetic field are distributed according to the normal law, then the conditional probability that the
measured value is equal to $\mathcal{B}^{\prime}$ for the star at the real value of $\mathcal{B}$ of the rms magnetic field for this star is equal to

$$
\begin{equation*}
P\left(\mathcal{B}^{\prime} \mid \mathcal{B}\right)=\frac{1}{\sqrt{2 \pi} \sigma} e^{-\frac{1}{2}\left(\frac{\mathcal{B}-\mathcal{B}^{\prime}}{\sigma}\right)^{2}} \tag{7}
\end{equation*}
$$

where $\sigma$ is the error of $\overline{\mathcal{B}}$ for the considered star. To calculate a value of $\sigma$, one can use formula (2).

The total probability that the rms magnetic field of the star with the number $i$ in the list of stars with the measured magnetic field accepts the value of $\mathcal{B}_{i}^{\prime}$ for the given parameters $\mathcal{B}$ and $\sigma_{\mathcal{B}}$ can be obtained by multiplying probabilities (6) and (7) and integrating over all possible values of $\mathcal{B}$ :

$$
\begin{array}{r}
P\left(\mathcal{B}_{i}^{\prime} \mid \overline{\log \mathcal{B}}, \sigma_{\overline{\log \mathcal{B}}}, \sigma_{i}\right)= \\
\frac{1}{2 \pi \sigma_{\log \mathcal{B}} \sigma_{i} \ln 10} \int_{\mathcal{B}_{\min }}^{\mathcal{B}_{\max }} e^{-\frac{1}{2}\left(\frac{\left(\log \mathcal{B}-\overline{\log \mathcal{B})^{2}}\right.}{\sigma_{\log \mathcal{B}}^{2}}+\frac{\left(\mathcal{B}-\mathcal{B}_{i}^{\prime}\right)^{2}}{\sigma_{i}^{2}}\right)} \frac{d \mathcal{B}}{\mathcal{B}} \tag{8}
\end{array}
$$

where for convenience we replace the integration over $\eta=$ $\log \mathcal{B}$ by integration over $\mathcal{B}$.

The integration limits of $\mathcal{B}_{\text {min }}$ and $\mathcal{B}_{\text {max }}$ are determined by the possible values of $\mathcal{B}$ for a given value of $\mathcal{B}_{i}^{\prime}$ (see, for example, Kholtygin et al., 2010a).

Let a set of $\left\{\mathcal{B}_{i}\right\}$ values of $r m s$ magnetic fields be defined for any group of stars. We introduce the likelihood function $\mathcal{L}$ as the product of probabilities (8) for all stars of a given group. Then

$$
\begin{equation*}
-\ln \mathcal{L}=\sum_{i=1}^{n}-\ln P\left(\mathcal{B}_{i}^{\prime} \mid \overline{\log \mathcal{B}}, \sigma_{\overline{\log \mathcal{B}}}, \sigma_{i}\right) \tag{9}
\end{equation*}
$$

where $n$ is the number of stars in the group.
The parameters of the distribution function $\eta=\overline{\log \mathcal{B}}$ and $\sigma_{\eta}=\sigma_{\log \mathcal{B}}$ of the rms magnetic fields for the group of stars under consideration can be found from the condition of the


Fig. 2. Distributions of the rms magnetic field (dotted line) for O stars (left) and for B stars (right). The solid line shows the log-normal fit.

Table 1. Average values of magnetic fields, magnetic fluxes, and their standard deviations for magnetic OBA stars, WM stars, normal pulsars, and magnetars.

| Stellar group | Number of stars | $\overline{\log (\mathcal{B})}$ | $\sigma_{\log \mathcal{B}}$ | $\overline{\log (\Phi)}$ | $\sigma_{\log \Phi}$ | Reference |
| :--- | :---: | ---: | :---: | :---: | :---: | :---: |
| O stars | 14 | $2.56 \pm 0.13$ | $0.48 \pm 0.10$ | $27.6 \pm 0.12$ | $0.62 \pm 0.01$ | Pres. paper |
| - | 10 | $2.62 \pm 0.16$ | $0.26_{-0.11}^{+0.24}$ | - | - | M21 |
| B stars | 141 | $2.70 \pm 0.04$ | $0.49 \pm 0.03$ | $26.6 \pm 0.04$ | $0.49 \pm 0.03$ | Pres. paper |
| - | 90 | $2.83 \pm 0.10$ | $0.65 \pm 0.09$ | - | - | M21 |
| A stars | 121 | $2.70 \pm 0.04$ | $0.44 \pm 0.03$ | $26.3 \pm 0.01$ | $0.47 \pm 0.01$ | Pres. paper |
| - | 93 | $3.06 \pm 0.11$ | $0.66 \pm 0.07$ | - | - | M21 |
| WM stars | 19 | $0.10 \pm 0.12$ | $0.51 \pm 0.09$ | $23.1 \pm 0.11$ | $1.23 \pm 0.09$ | Pres. paper |
| - | 5 | $0.15 \pm 0.50$ | $0.65_{-0.27}^{+0.57}$ | - | - | M21 |
| OB stars | 51 | $2.57 \pm 0.07$ | $0.51 \pm 0.05$ | $27.0 \pm 0.05$ | $0.70 \pm 0.04$ | Pres. paper |
| OBA stars | 276 | $2.69 \pm 0.03$ | $0.47 \pm 0.02$ | $26.5 \pm 0.02$ | $0.52 \pm 0.02$ | Pres. paper |
| Radio pulsars | 2061 | $12.09 \pm 0.01$ | $0.51 \pm 0.01$ | $24.1 \pm 0.03$ | $0.51 \pm 0.01$ | Pres. paper |
| Magnetars | 21 | $14.38 \pm 0.08$ | $0.53 \pm 0.07$ | $26.6 \pm 0.11$ | $0.41 \pm 0.07$ | Pres. paper |

M21 = Makarenko et al. (2021b)
maximum of the logarithm of the likelihood function (with the opposite sign) $-\ln \mathcal{L}$. Figure 2 shows the approximations of the magnetic field of O and B star distribution obtained by the least squares method. The similar approximations were obtained for other groups of stars. The error of the mean values of $\log \mathcal{B}$ and $\sigma_{\log \mathcal{B}}$ does not exceed 0.12 dex indicating a good quality of our approximations.

The average values of $\overline{\log (\mathcal{B})}$ and corresponding standard deviations $\sigma_{\log \mathcal{B}}$ for groups of $\mathrm{O}, \mathrm{B}$, and A stars as well as for all magnetic OBA stars and weakly magnetic stars are given in Table 1 (Columns 3 and 4). Column 2 of the table gives the numbers of stars with measured magnetic fields for these groups of stars. These values are compared to those obtained by Makarenko et al. (2021b).

The values of $\overline{\log (\mathcal{B})}$ and $\sigma_{\log \mathcal{B}}$ obtained by us slightly differ from those calculated by Makarenko et al. (2021b) due to an increasing number of objects in our samples of OBA and weakly magnetic stars. However, in all cases these differences do not exceed three standard deviations.

We also added to Table 1 the characteristics of the magnetic field and magnetic flux distributions for the ensemble of magnetic OB stars in which we included O and $\mathrm{B} 0-\mathrm{B} 2$ stars. These stars mostly explode as supernovae and then become neutron stars.

The last rows of the table contain the parameters of the magnetic field and magnetic flux distributions of neutron stars, separately for normal neutron stars (radio pulsars) and magnetars.

### 2.4 Magnetic flux distribution

The stellar magnetic fluxes with the known rms magnetic field $\mathcal{B}$ can be calculated using the following formula of Kholtygin et al. (2010a):

$$
\begin{equation*}
\Phi \approx 4 \pi \mathcal{B} R_{*}^{2} \tag{10}
\end{equation*}
$$

Stellar radii $R_{*}$ are taken from the papers cited in Sect. 2.2.
Calculating the magnetic fluxes of neutron stars (radio pulsars and magnetars), one should take into account the
difference between the surface magnetic field $B_{\mathrm{s}}$ defined by formula (4) and the rms magnetic field $\mathcal{B}$. According to Ferrario and Wickramasinghe (2006) (their formula (2)) and Kholtygin et al. (2010b) $B_{\mathrm{s}}=\Phi / \pi R_{*}^{2}$, i.e., $B_{\mathrm{s}} \approx 4 \mathcal{B}$. In this case, the following relation should be used for neutron stars:

$$
\begin{equation*}
\Phi=\pi B_{\mathrm{s}} R_{*}^{2} . \tag{11}
\end{equation*}
$$

The magnetic flux distributions are obtained in the same way as in the case of magnetic fields using the measured magnetic fields. Figure 3 (left) shows such distributions for OBA and weakly magnetic stars, as well as normal pulsars and magnetars.

The results of our analysis show that magnetic fluxes as well as rms magnetic fields can be described by a log-normal distribution. The distribution parameters for all considered groups of stars are determined by the procedure described in Sect. 2.3.

The parameters of the magnetic flux distribution for all analyzed groups of stars are given in Columns 5 and 6 of Table 1 . The last rows of the table contain the average magnetic fluxes and the corresponding standard deviations for radio pulsars and magnetars. The obtained by us approximations for WM stars and normal pulsars are illustrated in Fig. 4.

The mean magnetic fluxes for magnetars and OB stars are close as it was noted by Igoshev and Kholtygin (2011), Kholtygin and Makarenko (2019), Makarenko et al. (2020), Makarenko et al. (2021a), Makarenko et al. (2021b). Kholtygin and Makarenko (2019) noted that the magnetic fluxes of normal pulsars and weakly magnetic stars are close too.

Meanwhile, the average magnetic fluxes of radio pulsars obtained in this work exceed those for weakly magnetic stars by an order of magnitude. Perhaps, this excess is associated with increasing magnetic fluxes during the gravitational collapse of magnetic stars as a result of fast dynamo processes. We also note that the sample of weakly magnetic stars is still small, and such an excess at least partly may be a result of the sample scarcity.


Fig. 3. Distribution of magnetic fluxes (left) and magnetic moments (right) for OB stars, weakly magnetic stars, normal pulsars, and magnetars.


Fig. 4. Magnetic flux distributions (dashed line) of weakly magnetic stars (left) and normal pulsars (right). The log-normal fits are shown by the solid line.

### 2.5 Common magnetic fields and magnetic flux distributions for magnetic and weakly magnetic stars

The magnetic field and magnetic flux distributions presented in Sects. 2.3 and 2.4 refer separately for magnetic and weakly magnetic stars. At the same time, it seems a good idea to consider all stars of these spectral types as a common ensemble.

Suppose we know the magnetic field distribution for magnetic OBA stars $f_{\mathrm{M}}(\overline{\log \mathcal{B}})$ and the corresponding distribution for weakly magnetic stars $f_{\mathrm{WM}}(\overline{\log \mathcal{B}})$. Suppose the fraction of magnetic stars in a complete ensemble of stars of any type to be $\alpha$, then the fraction $1-\alpha$ refers to weakly magnetic stars. The complete distribution function of the magnetic fields of the entire ensemble of OBA stars can be expressed as follows:

$$
\begin{equation*}
f_{\mathrm{OBA}}(\overline{\log \mathcal{B}})=(1-\alpha) f_{\mathrm{WM}}(\overline{\log \mathcal{B}})+\alpha f_{\mathrm{M}}(\overline{\log \mathcal{B}}) \tag{12}
\end{equation*}
$$

The results of applying formula (12) to OBA and neutron star ensembles are illustrated in Fig. 5. The fraction of magnetars among the known neutron stars is $\sim 1 \%$. At the same time, due to the extremely short lifetime of magnetars and the difficulty in their detection as compared to the detection of radio pulsars the actual fraction of magnetars should be much larger. For this reason, in this and the following figure we use the same $\alpha$ parameter values for neutron stars as for OBA stars.

The figure shows a bimodal distribution of magnetic fields for these groups of stars. As can be seen from the figure, the distribution shapes for these different groups of stars are close, although the absolute values of $r m s$ magnetic fields differ by $\sim 12$ orders of magnitude.

Meanwhile, the difference between the maxima of the distribution of $r m s$ magnetic fields for magnetic and weakly magnetic OB stars, as well as for radio pulsars and magnetars, is $\sim 2.2-2.5$ dex. Almost the same difference between the


Fig. 5. Full distributions of the $r m s$ magnetic fields of OBA stars (left) and neutron stars (right) for various values of the parameter $\alpha$.


Fig. 6. The same as in Fig. 5 but for magnetic fluxes.
mean magnetic fields of normal pulsars and magnetars was obtained in Makarenko et al. (2021b). Thus, in the first approximation the distribution of neutron stars magnetic fields is the distribution of magnetic fields of OBA stars shifted by $\sim 12$ orders of magnitude.

An important feature of the distribution functions under consideration is the significantly greater height of the peak in the distribution of magnetic fields for weakly magnetic stars compared to magnetic stars. Meanwhile, the number of stars with measured magnetic fields in this region does not exceed two dozen. Thus, the region of $r m s$ magnetic fields up to several tens of gauss is the terra incognita of the physics of magnetic fields and is therefore of great interest for future researchers.

The common magnetic fluxes for OBA and neutron star distributions for various values of the parameter $\alpha$ are shown in Fig. 6. As well as for magnetic fields, the shapes of distributions are similar; however, in the case of magnetic flux distributions the distribution maxima are close. Such a hy-
pothesis was earlier proposed by Ferrario et al. (2009). In the meantime, the distribution of magnetic fluxes obtained in the cited paper (Fig. 4) corresponds to magnetic stars only. In the framework of the approach proposed in the present paper it is possible to describe the magnetic fields and magnetic flux distributions for both magnetic and weakly magnetic stars with unified function.

## 3 Magnetic moment distribition

An important characteristic of the global magnetic properties of stars is their magnetic moments $\mu$. According to Arge et al. (1995) the magnetic moment of a star with a polar field $B_{\mathrm{p}}$ and a radius $R_{*}$

$$
\begin{equation*}
\mu=\frac{1}{2} B_{\mathrm{p}} R_{*}^{3} . \tag{13}
\end{equation*}
$$



Fig. 7. The same as in Fig. 5 but for magnetic moments.

Table 2. Average magnetic moments and their standard deviations for OBA, WM stars, pulsars, and magnetars.

| Stellar group | Number of stars | $\overline{\log \mu}$ | $\sigma_{\log \mu}$ |
| :--- | :---: | :---: | :---: |
| O stars | 14 | $38.8 \pm 0.04$ | $0.62 \pm 0.04$ |
| B stars | 141 | $37.3 \pm 0.04$ | $0.55 \pm 0.03$ |
| A stars | 121 | $36.9 \pm 0.01$ | $0.51 \pm 0.01$ |
| WM stars | 19 | $33.6 \pm 0.14$ | $1.33 \pm 0.11$ |
| OB stars | 51 | $37.9 \pm 0.14$ | $0.89 \pm 0.12$ |
| OBA stars | 276 | $37.1 \pm 0.03$ | $0.62 \pm 0.02$ |
| Radio pulsars | 2061 | $29.9 \pm 0.01$ | $0.51 \pm 0.01$ |
| Magnetars | 21 | $32.1 \pm 0.02$ | $0.44 \pm 0.02$ |

To test the hypothesis that the magnetic moments of massive stars hold during their evolution from the main sequence to the formation of neutron stars, we calculate the magnetic moment distributions according to the method described in Sect. 2.3. The resulting distributions calculated according to the data cited in Sect. 2.2 are shown in Fig. 3 (right). An analysis of the figure shows that the average magnetic moments of normal pulsars and magnetars are $\sim 5$ orders of magnitude smaller than the corresponding values for their progenitors - weakly magnetic and magnetic massive stars, correspondingly.

Our analysis shows that the magnetic moment distributions can be described by a log-normal function as it is made for magnetic fields and magnetic flux distributions:

$$
\begin{equation*}
f\left(\log \mu \mid \overline{\log \mu}, \sigma_{\log \mu}\right)=\frac{1}{\sqrt{2 \pi} \sigma_{\log \mu}} e^{-\frac{1}{2}\left(\frac{\log \mu-\overline{\log \mu}}{\sigma_{\log \mu}}\right)^{2}} . \tag{14}
\end{equation*}
$$

Parameters of distribution (14) for all groups of stars considered by us are obtained by the same way as for rms magnetic fields and magnetic fluxes and given in Table 2. The common magnetic moment distributions for OBA and neutron stars are shown in Fig. 7.

## 4 Conclusions

The statistical properties of the rms magnetic fields, magnetic fluxes, and magnetic moments of OBA and neutron stars have been investigated from the analysis of the recent measurements of OBA stars magnetic fields, for both magnetic and weakly magnetic ones, and from the estimates of the neutron stars magnetic fields.

The distributions of the above-mentioned values are shown to be probably log-normal. The common distributions of these quantities for both magnetic and weakly magnetic stars are obtained.

We conclude that there is a common distribution for the magnetic fields of all OBA stars, both magnetic and weakly magnetic. Evidence is presented in favor of the earlier conclusion that magnetars are the descendants of magnetic OB stars, and weakly magnetic OB stars are the progenitors of normal pulsars.

The stellar magnetic moments are shown to decrease during the evolution of massive stars from the main sequence to the formation of neutron stars by on average 5 orders of magnitude.

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[^0]:    ${ }^{1}$ The term "magnetic stars" refers to chemically peculiar BA stars with a strong magnetic field of several hundred gauss or more. In this paper we put all OBA stars with a registered magnetic field $\mathcal{B}>$ 30 G to be magnetic. Since the term is used by us only in this sense, its extended interpretation should not cause misunderstanding.

