

On the possibility of prompt computing a preliminary orbit along a short arc for the purpose of conducting repeated observations on the current night

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ABSTRACT

The paper explores the conditions for a successful dynamic planning of the repeated observations of new space objects when the telescope operates in automatic mode. To organize the dynamic planning, we wrote software for determining the preliminary orbits and calculating the ephemeris for the near future in the format of a telescope operation scheduler. The computer program finds a preliminary orbit using the Laplace method and then improves it using the differential correction method. The program first uses the two-body problem formulas and then the SGP4 model to improve the orbit and calculate the ephemeris. This study considers 248 pairs of observation sets for 119 small-sized objects with an orbital semi-major axis from 7254 to 44 674 km, obtained during one night with the AT-64 telescope of the Crimean Astrophysical Observatory in 2022. Our software determined preliminary orbits by the first observation set and computed ephemeris for the time points of the second observation set for 234 pairs. Next, we estimated their quality by determining the angular distance ($O - C$) between the observed and calculated positions of the object. We accept that the preliminary orbit and ephemeris have a good quality if $(O - C) < 45'$, that is, the object falls within a field of view of the AT-64 telescope. As a result, we find that it is advisable to get the first observation set with a topocentric arc of more than 1° (and more than 5° in case of orbits with a semi-major axis of less than 20 000 km) and a duration of more than 3 minutes in order to successfully determine the preliminary orbit and compute a good-quality ephemeris during 60 minutes from the epoch of preliminary elements. But a repeated observation should be carried out within 30–40 minutes if objects have a high area-to-mass ratio.

Key words: space debris, Laplace method, SGP4, high area-to-mass ratio

1 Introduction

A large number of small-sized space objects of artificial origin are currently observed in near-Earth orbits. Further exploration of near-Earth space is impossible without knowledge of the current situation, analysis of sources and patterns of the evolution of space debris. The Crimean Astrophysical Observatory (CrAO) occupies one of the leading positions in this field, taking an active part in the space debris research program since 2003. The first results of studying fragments of space debris in the geostationary region are presented in [Volvach et al. \(2006\)](#).

Not all observed space debris objects have been cataloged; many of them are new or have been lost. To more accurately determine the orbital parameters of a new object when it is detected, it is advisable to obtain several sets of observations during one night. One can automate this process by determining a preliminary orbit from the first set and calculating the ephemeris for the near future in the format of a telescope operation scheduler. Such dynamic planning would allow repeated observations of the object to be conducted during the current night.

At the moment, a program for determining the preliminary orbit has been developed and is undergoing laboratory testing. Last year, a comparison was made between the Laplace methods and the method of apparent motion parameters (AMP) by determining the preliminary orbits of 2783 model objects with short sets of 6- and 8-min duration, containing 4 and 5 object positions with a 2-min interval, respectively. The results can be found in [Sannikova \(2022\)](#). In particular, it was found that the AMP method is often more accurate than the Laplace method, but the latter gives fewer failures and is more effective in extreme conditions, such as short observation arcs, highly elliptical and polar orbits, that is, the Laplace method is preferable in automatic operation conditions.

2 Methods and software implementation

This work examines 119 small-sized objects with an orbital semi-major axis from 7254 to 44 674 km, which were observed at the Crimean Astrophysical Observatory with the AT-64 telescope during 2022. For these objects, 248 pairs

of observation sets obtained on one night were selected. We also used data on orbital parameters and area-to-mass ratios (AMR) obtained at the Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences (KIAM RAS) for these objects (Fig. 1). Most of the objects considered are in the region of geostationary orbits (almost circular orbits with a semi-major axis of more than 40 000 km and a small inclination to the equatorial plane) and Molniya orbits (highly eccentric orbits with a large semi-axis of about 25 000 km and an inclination of about 65°).

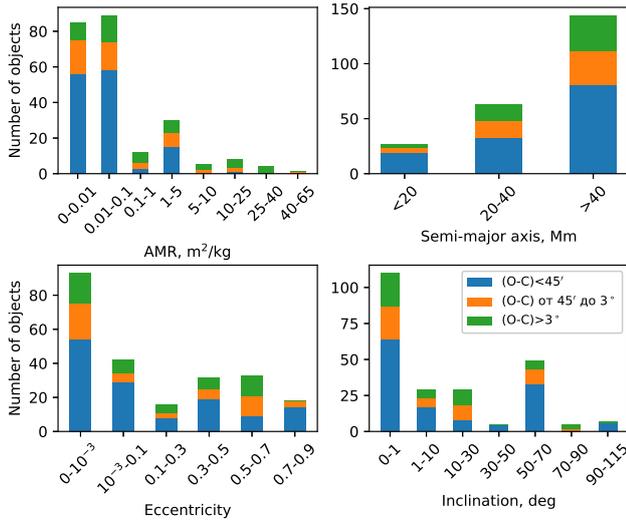


Fig. 1. Distribution in terms of AMR and orbital elements of 234 objects with successful calculation of the ephemeris.

Preliminary orbits are determined based on the first set of observations using the Laplace method, and then they are refined using the differential correction method.

Next, the ephemeris is calculated for the time points of the second set, and their accuracy is assessed. For this purpose, the angular distance ($O - C$) between the calculated and observed positions is determined. It is accepted that the preliminary orbit has a good quality if $(O - C) < 45'$, that is, the object falls within a field of view of the AT-64 telescope, an average quality at $45' \leq (O - C) \leq 3^\circ$ (the object falls within the field of view of the survey telescope), and a poor quality for $(O - C) > 3^\circ$.

The orbit is improved first within the framework of the unperturbed motion model using the formulas of the two-body problem and then using the Simplified General Perturbations model SGP4 (Vallado et al., 2006). If the refinement using the SGP4 model is successful, then the ephemeris is also calculated using the SGP4 model.

The Laplace and differential correction methods are implemented in Python in accordance with the techniques described in Bykov, Kholshevnikov (2013) and Escobal (1970). Determination of the observer's vector, as well as coordinate and time transformations, are performed using the basic package for astronomy Astropy¹, developed by the Astropy

community. The coordinates and velocities of objects at given time points are calculated either using the formulas of unperturbed motion within the framework of the two-body problem or using the `sgp4`² library, implementing a simplified model of the perturbed motion of near-Earth objects. All mathematical operations are carried out using the `numpy`³ package, which allows one to process vectors and matrices.

3 Results and discussion

Calculations have been carried out for 248 pairs of sets (hereinafter referred to as objects). The preliminary orbit was determined to be hyperbolic in 6 of these cases, and no further work was carried out. For another 8 cases, the position of the object, calculated from the preliminary orbit at the moments of the second observation set, turned out to be under the surface of the Earth, and, as a result, the ephemeris could not be calculated. In these 8 cases, the time interval between sets exceeded 80 minutes. The short topocentric arc of the first set of observations is a likely cause of these 14 failures, since it is less than 0.65° in these cases. For the remaining 234 objects, the preliminary orbit was successfully determined and the ephemeris was calculated for the moments of the second set, while:

- orbit improvement failed to be performed for 44 objects, of which for 16 – $(O - C) < 45'$, for 11 – $45' \leq (O - C) \leq 3^\circ$, and for 17 – $(O - C) > 3^\circ$;
- the orbit was improved only within the framework of the unperturbed motion model for 24 objects, of which for 5 – $(O - C) < 45'$, for 10 – $45' \leq (O - C) \leq 3^\circ$, and for 9 – $(O - C) > 3^\circ$;
- the ephemeris was calculated using the SGP4 model for 166 objects, of which for 112 – $(O - C) < 45'$, for 30 – $45' \leq (O - C) \leq 3^\circ$, and for 24 – $(O - C) > 3^\circ$.

Figure 1 shows the distribution of 234 objects along the AMR, semi-major axis, eccentricity and inclination, as well as the quantitative ratio of cases in which the angular distance ($O - C$) between the observed and calculated positions was less than $45'$, from $45'$ up to 3° , and more than 3° .

Figure 2 shows the angular distance ($O - C$) as a function of time between the epoch of the preliminary elements and the time points of the second set. The dots mark the beginning of the second set; the gray lines connect the beginning of the second set to the last observation of the current night. We can see that the number of cases with an angular distance greater than 3° grows with increasing the start time of the second set. Thus, the sooner the second observation is made, the greater the likelihood that the object will fall into the telescope's field of view. In addition, in Fig. 2 we can see that the poor quality of the ephemeris is most often obtained if the topocentric arc of the first set is shorter than 1° .

Let's take a closer look at the objects with different semi-major axes. For the objects with a semi-major axis less than 20 000 km (27 objects), the first set often contains a large number of points and has a long topocentric

¹ <http://www.astropy.org>

² Brandon Rhodes, <https://pypi.python.org/pypi/sgp4>

³ <https://numpy.org>

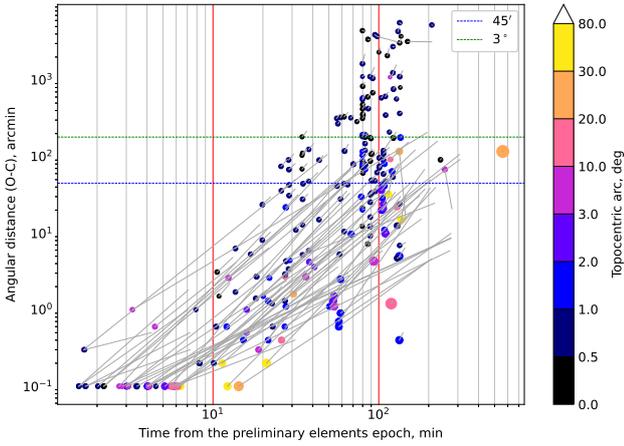


Fig. 2. The angular distance ($O - C$) between the observed and calculated positions as a function of the time interval between the epoch of the preliminary elements and the time points of the second set. The color of the marker indicates the length of the topocentric arc of the first observation set; the size of the marker is proportional to the duration of the first set.

arc (Figs. 3 and 4, top panel). In some cases, the preliminary orbit is determined so precisely that the angular distance between the observed and calculated positions is less than $45'$ even on the next turn. However, when the duration of the first set is less than 1 min and the arc is shorter than 2° , the preliminary orbit has a poor quality. It is advisable to obtain an arc longer than 5° and a duration greater than 3 min to ensure a good quality of the ephemeris.

For the objects with a large semi-axis from 20 000 to 40 000 km (63 objects), the first observation set with a very short topocentric arc (less than 1°) and a duration of less than 3 min is not rare. In these cases, the ephemeris is often of poor quality (see Fig. 3, middle panel). But if repeated observations are carried out within 1 hour after determining the preliminary elements, then the object most likely falls into the field of view of the telescope.

Most of the first observation sets of objects with a semi-major axis greater than 40 000 km (144 objects in total) also have a very short topocentric arc (Figs. 3 and 4, bottom panel). In Fig. 3 we can see that obtaining an accurate long time ephemeris is almost impossible if the first observation set has an arc of less than 0.5° and a duration of less than 2 min. The probability of being in the telescope's field of view is about 50% if the first set has a duration of 2–3 minutes and an arc of up to 1° . The quality of the ephemeris grows with increasing observation time and topocentric arc. In general, in most cases, repeated observations can be carried out the next 60–80 minutes after the epoch of preliminary elements, but it is desirable to have an arc of more than 1° .

The dependence of the ephemeris quality on the area-to-mass ratio of an object is shown in Fig. 4. Most of the objects considered have low AMR (Fig. 1). For them, the resulting ephemeris is often of good quality for 1–2 hours. There are a few objects with AMR of $1\text{--}25\text{ m}^2/\text{kg}$ for which the ephemeris has a good quality for 60–90 min. Unfortunately, the time interval between sets is 60 minutes or more for all objects with an AMR of more than $25\text{ m}^2/\text{kg}$, and the

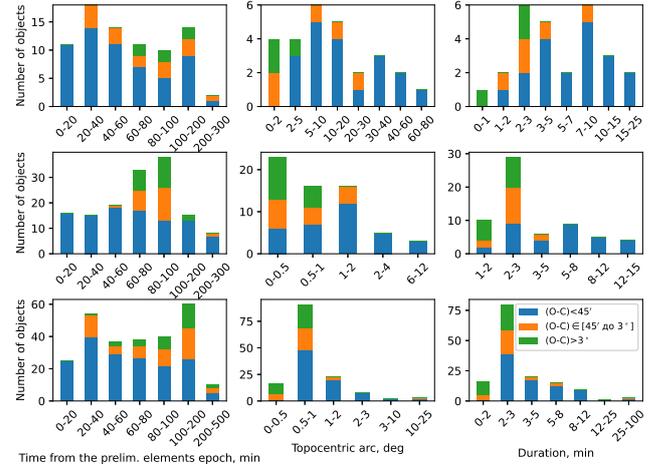


Fig. 3. Distribution of objects depending on the time interval between the epoch of preliminary elements and the time points of the second set, as well as on the characteristics of the first set: topocentric arc and duration. The upper panel refers to the objects with an orbital semi-major axis of less than 20 000 km; the middle, from 20 000 to 40 000 km; the lower, more than 40 000 km.

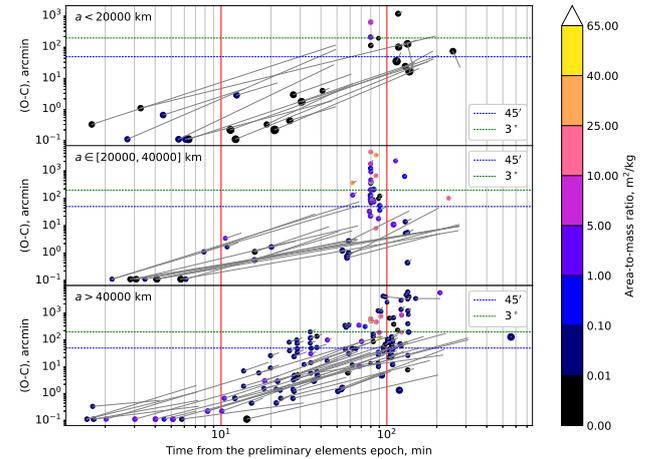


Fig. 4. Dependence of the quality of the ephemeris on the area-to-mass ratio of the object. The color of the marker indicates the value of the object's AMR; the size of the marker is proportional to the length of the topocentric arc of the first set.

ephemeris calculated for them has a poor quality. Since the objects with high AMR are most often lost, repeated observations must be carried out within 30–40 minutes after obtaining preliminary elements for the successful re-detection of such objects in automatic operation mode.

4 Conclusion

It is advisable to get the first observation set with a topocentric arc of more than 1° (and more than 5° in case of orbits with a semi-major axis of less than 20 000 km) and a duration

of more than 3 minutes in order to successfully determine the preliminary orbit and compute a good-quality ephemeris during 60 minutes from the epoch of preliminary elements. But a repeated observation should be carried out within 30–40 minutes if objects have a high area-to-mass ratio.

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