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Long-term radio variability properties of an extensive sample of bright blazars

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ABSTRACT

The RATAN-600 multi-frequency catalog of blazars (BLcat), which combines radio data for more than 1800 blazars selected from the Roma-BZCAT catalog, is available in the interactive form on the SAO RAS home page. The blazars were observed with the RATAN-600 telescope in 2005–2023 quasi-simultaneously at four to six frequencies in the range 1.2–22.3 GHz. We present the radio variability properties of different subclasses of blazars presented in BLcat based on the RATAN-600 long-term measurements. Our results support previous studies of blazar radio variability showing that the variability index tends to increase with frequency and that the BL Lacs subclass is more variable than others. We show an impact of the monitoring cadence on the variability estimates. The use of time series with longer monitoring periods, more regular observations, and more data points is crucial to detect blazar activity and obtain its characteristics.

Key words: active galactic nuclei, blazars, radio continuum, catalogs

1 Introduction

Radio variability plays an important role in understanding the processes taking place in active galactic nuclei (AGNs). In order to study it thoroughly, it is crucial to collect the data over a long period of time, in a monitoring lasting decades. Since this is a very time-consuming task, only a few observatories have been able to observe AGNs continuously, e.g., the Owens Valley Radio Observatory at 15 GHz [\(Richards et al.,](#page-3-1) [2011\)](#page-3-1), the Metsähovi Radio Observatory at 22 and 37 GHz [\(Teräsranta et al.,](#page-3-2) [2005;](#page-3-2) [Nieppola et al.,](#page-3-3) [2009\)](#page-3-3), the University of Michigan Radio Astronomy Observatory at 4.8, 8, and 14.5 GHz [\(Aller et al.,](#page-3-4) [1985\)](#page-3-4), and the Crimean Astrophysical Observatory at 22.2 and 36.8 GHz [\(Nesterov et al.,](#page-3-5) [2000;](#page-3-5) [Volvach et al.,](#page-3-6) [2009\)](#page-3-6) are among the most notable ones. At other radio observatories, however, light curves even for wellknown AGNs and blazars are sparsely sampled because they have been observed more or less intensively mainly during flare activities. Another common concern is the availability of only one frequency at a time for observations at the telescope. Given everything stated above, the RATAN-600 blazar observing program, which has been carried out regularly since the early 2000s, makes a valuable contribution to the study of broadband radio variability with its long-term monitoring in the 1–22 GHz frequency range. In addition, the distinctive design of the antenna makes it possible to obtain quasi-simultaneous measurements at five to six frequencies [\(Korolkov, Pariiskii,](#page-3-7) [1979;](#page-3-7) [Sotnikova,](#page-3-8) [2020\)](#page-3-8).

About a decade has passed since the publication of the first version of the BL Lac blazar catalog that contained a few hundred objects, mostly of the BL Lacertae type [\(Min](#page-3-9)[galiev et al.,](#page-3-9) [2014\)](#page-3-9). We continue to extend the source list, adding new types of blazars and developing online tools that help to estimate the radio properties of the sources. We have recently released an updated version of the catalog – BLcat^{[1](#page-0-1)}. It contains a collection of long-term (∼20 years), quasi-simultaneous six-frequency (1.2–22.3 GHz) flux density measurements for more than 1800 blazars of different types [\(Sotnikova et al.,](#page-3-10) [2022\)](#page-3-10). The BLcat also provides online access to the useful scientific tools for a more detailed analysis of the source properties: we additionally supplement the radio data from external sources via the Astronomical CATalogs Support System (CATS^{[2](#page-0-2)}) database [\(Verkhodanov](#page-3-11) [et al.,](#page-3-11) [1997,](#page-3-11) [2005\)](#page-3-12) to build light curves and radio spectra in a wider frequency range, and also we build in radio variability and spectral index calculation tools.

In this short report we provide observational statistics and present radio variability measurements at six frequencies for different blazar types presented in the catalog and obtained using the online tools of the BLcat catalog.

¹ <https://www.sao.ru/blcat/>

² <https://www.sao.ru/cats/>

2 The concept of the catalog

Blazars of the BLcat catalog are selected from the 5th Edition of the Roma-BZCAT catalog [\(Massaro et al.,](#page-3-13) [2015\)](#page-3-13) with the following criteria: a source declination between −34◦ and +49° and with an NVSS flux density limit of $S_{1,4} \ge 100$ mJy. The construction of the Roma-BZCAT sample of blazars is inhomogeneous, e.g., it is not based on a flux-limited all-sky survey, but it represents a comprehensive list of well-known, carefully checked, multi-band observed blazars. Since that we have chosen it to form a source list for our regular blazar monitoring program at RATAN-600 and to further build up our own RATAN-600 multi-frequency observation based catalog of blazars.

The catalog contains 1831 blazars of four types, classified according to [Massaro et al.](#page-3-13) [\(2015\)](#page-3-13) as follows: 547 BL Lac objects (BZB) and BL Lac objects with a significant dominance of the galaxy emission in their SED (BZG), 1151 Flat Spectrum Radio Quasars (BZQ), and 133 blazars of uncertain type (BZU) having peculiar characteristics but also exhibiting blazar activity.

The catalog presents irregular time series. The number of observations varies from one to more than one hundred and twenty epochs with an average value of 18. For most blazars (66%) there are no more than five epochs of observations. For 9% of the blazars there are 15–49 epochs of observations; for 20% of the blazars, 50–70 epochs of measurements; for 5% , 70–115 epochs.

Fig. 1. Distribution of the number of RATAN-600 measurements Nobs for the blazars in BLcat.

The distribution of the number of measurements that we obtained for all BLcat blazars is shown in Fig. [1.](#page-1-0) The peak around N_{obs} \sim 5 is due to a lack of time to observe each blazar in a relatively short period, and the second peak around $N_{obs} \sim 60$ is formed by well-known regularly monitored blazars, usually bright in the radio band and gamma-ray emitting sources.

3 Long-term radio variability properties of blazars

Unlike other blazar catalogs that contain only one or two radio measurements, e.g., CRATES [\(Healey et al.,](#page-3-14) [2007\)](#page-3-14) and BROS [\(Itoh et al.,](#page-3-15) [2020\)](#page-3-15) among the most known ones, we collect long-term radio data at six frequencies. This allows us to study the broadband radio properties of the blazars using the catalogs build-in online tools to construct radio spectra, light curves, to measure spectral indices or calculate flux density variability. We have used the variability index parameter [\(Aller et al.,](#page-3-16) [1992\)](#page-3-16) to estimate the peak-to-peak flux density variations of the blazars:

$$
V_S = \frac{(S_{\text{max}} - \sigma_{S_{\text{max}}}) - (S_{\text{min}} + \sigma_{S_{\text{min}}})}{(S_{\text{max}} - \sigma_{S_{\text{max}}}) + (S_{\text{min}} + \sigma_{S_{\text{min}}})},
$$
(1)

where S_{max} and S_{min} are the maximum and minimum flux densities over all epochs of observations; $\sigma_{S_{\text{max}}}$ and $\sigma_{S_{\text{min}}}$ are their uncertainties.

Table 1. The median and mean values of the variability index obtained for four types of blazars at six RATAN-600 frequencies. The standard deviations for mean values are given in parentheses.

	N	median	mean	ν , GHz
BZB	166	0.34	0.35(0.20)	
BZG	13	0.29	0.25(0.18)	22.3
BZO	538	0.28	0.29(0.17)	
BZU	63	0.28	0.30(0.19)	
BZB	228	0.29	0.30(0.19)	
BZG	27	0.16	0.23(0.17)	11.2
BZQ	581	0.25	0.25(0.16)	
BZU	81	0.25	0.27(0.19)	
BZB	207	0.16	0.21(0.17)	
BZG	17	0.10	0.12(0.08)	8.2
BZO	400	0.09	0.13(0.13)	
BZU	61	0.13	0.16(0.13)	
BZB	255	0.25	0.28(0.20)	
BZG	29	0.25	0.28(0.22)	4.7
BZO	608	0.20	0.23(0.19)	
BZU	89	0.22	0.26(0.22)	
BZB	115	0.18	0.20(0.15)	
BZG	10	0.25	0.27(0.14)	2.3
BZQ	386	0.13	0.14(0.11)	
BZU	46	0.17	0.16(0.13)	
BZB	54	0.20	0.25(0.17)	
BZG	6	0.23	0.24(0.09)	1.2
BZO	115	0.21	0.22(0.15)	
BZU	14	0.23	0.30(0.17)	

All four types of blazars demonstrate the highest variability levels at 22.3 GHz (median $V_S \sim 0.28-0.34$) and the lowest ones at 8.2 GHz (median $V_S \sim 0.09-0.16$). Regarding the former, it should be noted that the measurements at 8.2 GHz are significantly affected by the considerable incompleteness of the observations since only one radiometric complex is equipped with this frequency receiver. In general, there is a tendency for the variability index to increase toward higher Long-term radio variability properties... 7

frequencies for all blazars [\(Tornikoski et al.,](#page-3-17) [2000\)](#page-3-17). The variability parameters of the subsample of BZU blazars suggest that they are closer to the BL Lac blazar type, being slightly more variable than BZQs. Two subsamples of BL Lac blazars show different behavior across the frequency range: blazars with galaxy domination are more variable at low frequencies, while BL Lacs without significant host galaxy contribution are more variable at high frequencies, also being the most variable subsample among four at 4.7–22.3 GHz. The revealed inconsistency of an average variability for BZG and BZB blazars might be affected by the small number of BZGs in the sample. Alternatively, we consider that the prevailing emission from the jet in BZBs causes their high variability amplitudes at high frequencies, while the influence of the presence of an interstellar matter in a host galaxy in BZGs contributes significantly at low frequencies. The median and mean values of V_s at six frequencies estimated for four types of blazars are shown in Table [1.](#page-1-1)

The blazars of BLcat are located at different distances with the redshift values ranging from 0.005 up to 5.285; hence the variability indices measured at a given observed frequency (v_{obs}) have a large scatter of the source's rest frame frequency ($v_{\text{rest}} = (1+z)v_{\text{obs}}$), representing different emission regions in a source. We plot the variability index distribution against both v_{obs} and v_{rest} in Fig. [2.](#page-2-0) We found a fairly wide scatter of the variability index throughout entire range of 1– 130 GHz rest frame emission frequencies. Notably, there is a concentration of sources with $V_s \geq 0.8$ from about 4.7 GHz to 30 GHz in the rest frame, which reflects $v_{\text{obs}} = 4.7 \text{ GHz}$ as a frequency with the largest number of observations. The emission frequencies above ∼ 10 GHz are associated with the dominating core component; and below 10 GHz, with a combination of core and extended components. For the 210 blazars with unknown redshifts, v_{rest} and the corresponding V_S were not calculated, and these 210 sources are not shown in Fig. [2.](#page-2-0)

Fig. 2. The rest frame v_{rest} frequency distribution of the variability indices V_S calculated at the corresponding RATAN-600 frequencies $v_{\text{obs}} = 1.2 - 22.3 \text{ GHz}$ shown on the upper axis.

We selected 32 blazars with a time scale of RATAN-600 observations of at least 10 years and a sufficient number of

observation epochs $N_{\text{obs}} \geq 70$ to more reliably assess the variability for the longest monitoring subsample. Ten years in the observer's frame correspond to 1.5 years in the source's rest frame for the most distant objects $(t_{\text{rest}} = t_{\text{obs}}/(1 + z))$. And since blazars show the variability of their radio emission on scales from a few months to several years, a scale of 1.5 year is sufficient for an objective assessment of the time scale of the variability. Based on the radio data provided by BLcat for these 32 blazars, we calculated the following average values obtained with RATAN-600: $N_{\text{obs}} = 85$, $S_{4.7} = 1.35$ Jy, $t_{\rm obs}$ = 15.5 yrs, $t_{\rm rest}$ = 11.2 yrs; and the variability index is estimated as $V_{S_{22,3}} = 0.53$, $V_{S_{11,2}} = 0.47$, $V_{S_{8,2}} = 0.34$, $V_{\text{S}_{4,7}} = 0.43, V_{\text{S}_{2,3}} = 0.32,$ and $V_{\text{S}_{1,2}} = 0.28$. These values of the variability index are considerably higher than those for the whole sample [\(Sotnikova et al.,](#page-3-10) [2022\)](#page-3-10), which is an evident consequence of continuous monitoring in the case of a more representative subsample of 32 blazars. It demonstrates the importance of continuous high-cadence long-term monitoring to reveal flux density variations in blazars.

4 Discussion

The main results obtained are in good agreement with previous studies of the variability of AGNs and blazars. They, in common, reflect the fact that BL Lac objects generally have a greater contribution from compact jet emission to the total flux density and are therefore more variable at the studied frequencies than the flat-spectrum quasars. Additionally, the variability evolves with frequency in accordance with a moving shock in the jet model [\(Marscher, Gear,](#page-3-18) [1985\)](#page-3-18).

Our findings are consistent with those of [Aller et al.](#page-3-19) [\(1999\)](#page-3-19), who found that among radio-loud AGNs the BL Lacs are more variable than quasars over a long-term dataset of 16 years at 4.8, 8, and 14.5 GHz . The average variability index amounts to 0.27 and 0.56 at 4.8 and 14.5 GHz, respectively.

[Kovalev et al.](#page-3-20) [\(2002\)](#page-3-20) studied the broadband radio spectra evolution for a large sample of 550 AGNs and found that the mean variability index decreases with decreasing frequency (in the same frequency range 1–22 GHz as we analyzed), which is in accordance with predictions of the model where a shock wave passes through a conical adiabatically expanding jet [\(Marscher, Gear,](#page-3-18) [1985\)](#page-3-18).

[Ciaramella et al.](#page-3-21) [\(2004\)](#page-3-21) also report a statistically significant difference in the distribution of the variability index for BL Lac objects compared to flat-spectrum radio quasars, with the BL Lac blazars appearing to be more variable at six frequencies (4.8–37 GHz). The authors find the correlation of the variability index with frequency, similar to what we showed in our work. The mean values of variability indices obtained for BL Lacs are ∼0.59 (at 4.8 GHz) and ∼0.74 (at 37 GHz), and for FSRQs they are ∼0.38 (at 4.8 GHz) and ∼0.61 (at 37 GHz). These values are higher compared to ours because their sample is significantly smaller and contains well-known variable blazars.

A number of valuable long-term variability studies for AGNs from the Planck catalog were carried out at 22 and 37 GHz using the 22-meter radio telescope of the Crimean Astrophysical Observatory. [Volvach et al.](#page-3-22) [\(2016a\)](#page-3-22) have estimated the variability indices at 37 GHz for 104 bright AGNs (mainly BL Lacs and quasars) to attain the value 0.6 within the time intervals of up to 15 years. This is noticeably higher than the variability estimated at longer radio wavelengths (e.g., [Volvach et al.](#page-3-23) [2012;](#page-3-23) [Vol'vach et al.](#page-3-24) [2014\)](#page-3-24). The authors suggest that the increase in variability indices at millimeter wavelengths is consistent with the hypothesis that the brightest AGNs are close supermassive black hole binaries at the evolutionary stage close to coalescence. The GHz-peaked spectrum (GPS) sources were found to be not as strongly variable as the other types of AGNs, with estimated variability indices of around 0.3 at 37 GHz in [Volvach et al.](#page-3-25) [\(2016b\)](#page-3-25).

5 Conclusions

We have studied the radio variability properties for a list of 1831 bright blazars using long-term measurements made quasi-simultaneously at six frequencies (1.2, 2.3, 4.7, 8.2, 11.2, and 22.3 GHz) with the RATAN-600 radio telescope. The flux density variations range from a few to 60–70% for individual sources. We found that the variability index increases with frequency, reaching an average value of about 0.3 at 22.3 GHz for the whole sample. This observational fact could be explained by the radiation from an expanding cloud of relativistic particles [\(van der Laan,](#page-3-26) [1966\)](#page-3-26). Starting at high radio frequencies, the flare spreads to lower frequencies as its amplitude decreases [\(Marscher, Gear,](#page-3-18) [1985\)](#page-3-18). BL Lac blazars found to be more variable at the frequencies higher than 4.7 GHz, while blazars with significant host galaxy contribution show a high variability index at 1.2–2.3 GHz relative to other subsamples; however, the number of objects in that BZG subsample is quite small.

We reveal an extreme radio variability of blazars by studying their broadband light curves obtained over a long time period with good time resolution. Thus, blazars observed for more than 10 years with a number of measurements $N_{\text{obs}} \geq 70$ show a variability index greater than 0.4 and 0.5 at 4.7 and 22.3 GHz, respectively.

The study demonstrates the importance of long-term radio monitoring of blazars with a high time cadence over a wide frequency range for a better understanding of physical processes occurring in their kpc-scale central parts.

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