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Boundary conditions and MHD equilibrium of force-free magnetic flux ropes

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ABSTRACT

We perform an analysis of the magnetohydrodynamic (MHD) equilibrium of force-free magnetic flux ropes taking into account boundary conditions. It was shown that the use of the generating function to solve the MHD equilibrium equation of a force-free flux rope can lead to incorrect conclusions. The continuity condition of the tangential component of the electric field on the boundary surface in the ideal magnetohydrodynamics approximation is always satisfied and does not require separate consideration.

Key words: Sun, coronal loops, kink instability

1 Introduction

Magnetic flux tubes (ropes) are one of the most important issues in cosmic magnetohydrodynamics (MHD). These are structures of cylindrical shape, with basic parameters (temperature, plasma density, magnetic field strength, and direction) being sharply changed in the vicinity of a hypothetical boundary. In particular, they are closely related to the MHD dynamo theory, as the buoyancy force is believed to be responsible for the emergence of a new magnetic flux from the convective zone onto the surface of the Sun and stars. These tubes can play a significant role in the process of star formation as well as solar and stellar activity.

The significance of studying solar coronal loops cannot be overestimated due to their relation with coronal heating, solar wind, flares, and coronal mass ejections. Therefore, it is not surprising that a large number of studies are dedicated to their exploration (see, e.g., Reale, 2014). Since these magnetic structures have quite sharp lateral boundaries, the issue on boundary conditions describing the linkage of magnetic tube parameters (often neglecting loop curvature) on different sides of the boundary is crucial when considering their MHD equilibrium. These conditions are commonly reduced to the equality of total pressure, and in the case of the perturbed boundary, to the equality of transverse velocity components on its inner and outer surfaces. Note that this approach is widely used in describing MHD phenomena not only in conditions of solar (Bennett et al., 1999; Carter, Erdélyi, 2008; Erdélyi, Fedun, 2010; Zaqarashvili et al., 2010, 2014; Ruderman, Terradas, 2015) but also laboratory plasma (Goedbloed, Poedts, 2004).

In the first case, the sharp boundary approximation naturally arises from the condition of high plasma conductivity and, consequently, a small thickness of the diffusion boundary layer formed as a result of the "blurring" of the magnetic field in the plasma. This narrow layer can play an extremely important role both in the MHD equilibrium of flux ropes and in satisfying the continuity condition of the tangential component of the magnetic field (Tsap, Shakhovskaya, 2000).

Solov'ev, Kirichek (2021) recently noted that in the paper of Tsap et al. (2020), dedicated to the MHD stability of a shielded (external azimuthal magnetic field $B_{\varphi e}(r > a) = 0$, where a is the cross-section radius) force-free flux rope, the adopted model is not well justified. This follows from the continuity of the tangential component of the electric field $\mathbf{E}_\tau$, implying that, according to Solov'ev, Kirichek (2021), a jump in the electric current at the boundary is impossible. Additionally, based on the solutions of the MHD equilibrium equation provided by the generating function, Solov'ev, Kirichek (2021) concluded that at the characteristic distance r_0 from the flux rope axis, a singular magnetic surface may form, where the current density j and the force-free field parameter α values grow infinitely, along with discontinuities in the derivatives $\partial j/\partial r$ and $\partial \alpha/\partial r$.

Despite we have no doubt that the continuity condition of the tangential component of the electric field at the tube boundary should be satisfied (Landau, Lifshitz, 1966), we cannot agree with the conclusions drawn by Solov'ev, Kirichek (2021). In our opinion, the authors did not take into account several circumstances that will be considered below.

2 Force-free magnetic flux rope with a sharp boundary

By using the cylindrical coordinate system (r, φ, z) and subscripts i and e for internal and external parameters, we perform an analysis of the MHD equilibrium of a straight axisymmetric flux rope with a magnetic field (see Fig. 1):

$$\mathbf{B} = \begin{cases} (0, B_{\varphi i}(r), B_{zi}(r)), & r \leqslant a; \\ (0, B_{\varphi e}(r), B_{ze}(r)), & r > a. \end{cases} \tag{1}$$

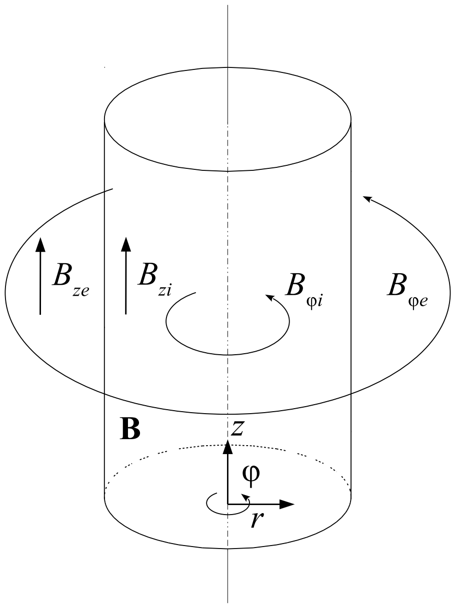

Fig. 1. Schematic representation of a magnetic flux rope.

As follows from expression (1), we consider the cylinder boundary as an MHD discontinuity. This approach allows one to significantly simplify the study of equilibrium magnetic configurations. Therefore, it is not surprising that model (1) is widely used by many authors (Bennett et al., 1999; Carter, Erdélyi, 2008; Erdélyi, Fedun, 2010; Zaqarashvili et al., 2010, 2014; Ruderman, Terradas, 2015) primarily for solar coronal flux ropes. Meanwhile, Solov'ev, Kirichek (2021) concluded on the insufficient justification of the model for a shielded magnetic flux rope ($B_{\varphi e}(r > a) = 0$) as well as for a laboratory pinch ($B_{\varphi e}(r > a) \propto 1/r$) with a sharp boundary due to the discontinuity in the tangential component of the electric field $\mathbf{E}_\tau$, implying that additional analysis should be carried out. Besides, Solov'ev, Kirichek (2021) pointed out the existence of special magnetic surfaces inside the equilibrium flux rope. To better understand the issue, let us gain insights into the mentioned reasonings.

Solov'ev, Kirichek (2021) obtained the dependences of the magnetic field components $B_z(r)$ and $B_\varphi(r)$ on the cross-section radius r of a straight axisymmetric force-free magnetic flux rope using the MHD equilibrium equation

$$\frac{d}{dr}\left(\frac{B_z^2 + B_\varphi^2}{8\pi}\right) + \frac{B_\varphi^2}{4\pi r} = 0. \tag{2}$$

The solution of equation (2) was found with the generating function

$$F(r) = B_z^2(r) + B_\varphi^2(r).$$

The function $F(r)$ allows expressing the magnetic field components as follows:

$$B_z(r) = \sqrt{F + (r/2)dF/dr}, \quad B_\varphi(r) = \sqrt{-(r/2)dF/dr}.$$

Thus, by selecting the function $F(r)$, taking into account the boundary conditions, we can determine $B_z(r)$ and $B_\varphi(r)$.

Constructing $F(r)$ is based on Ampere's law, which can be represented as

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{j}. \tag{3}$$

Integrating equation (3), for the azimuthal component of the magnetic field B_φ of a straight axisymmetric flux rope, we have

$$\oint B_\varphi dr = \frac{2}{c}\int j_z d\Sigma = \frac{2I_z}{c}, \tag{4}$$

where Σ is the cross-section of a flux rope. Equation (4) implies that the total longitudinal electric current $I_z = 0$ for a shielded magnetic flux rope (this case corresponds to neutralized current in the photosphere). Therefore, the current j_z must change its direction into opposite at some distance from the tube axis, and the azimuthal component of the magnetic field in the external region $B_\varphi(r > a) = 0$.

Taking into account all the mentioned above, Solov'ev, Kirichek (2021) defined the generating function as follows:

$$F(r) = B_0^2[G + (1 - G)f(r)], \quad G = \frac{B_{ze}^2}{B_0^2}, \tag{5}$$

where B_0 is the magnetic field on the flux rope axis, $f(r)$ is a certain continuous dimensionless decreasing function describing the distribution of the equilibrium magnetic field inside a flux rope. Note that the radius of the shielded flux rope a was determined as

$$F(a) = B_{ze}^2, \tag{6}$$

i.e., by the point $r = a$ at which $B_\varphi(a) = 0$. Condition (6), according to equation (5), implies that $f(a) = 0$, although the functions $f(r)$ invoked by Solov'ev, Kirichek (2021) tend to zero only asymptotically (e.g., ($f(r) = \exp(-k^2r^2)$). This means that the radius of the magnetic flux rope a is not defined since it can be considered infinitely large. Consequently, the shielding region of a flux rope is also formally unlimited (see also Solov'ev, 2022). The approach used by the authors is explained by their aim to avoid approximating a sharp boundary of "plasma – plasma", which, in their opinion, violates the continuity condition of the tangential component of the magnetic field $\mathbf{E}_\tau$.

In our view, it would be more reasonable to identify the characteristic radius of the flux rope a with the boundary layer thickness Δr, where the most abrupt changes in the main parameters of the flux rope occur, including the electric current j and the parameter α. In this case, if $\Delta r/a \longrightarrow 0$, then such a boundary region is reduced to the boundary lateral surface, which can be considered as an MHD discontinuity, thus avoiding the appearance of any peculiarities and inconsistencies in solving the MHD equilibrium equation (2). However, in general, Δr can vary within wide ranges.

We believe that under the correct statement of the problem, there should be no special (singular) magnetic surfaces

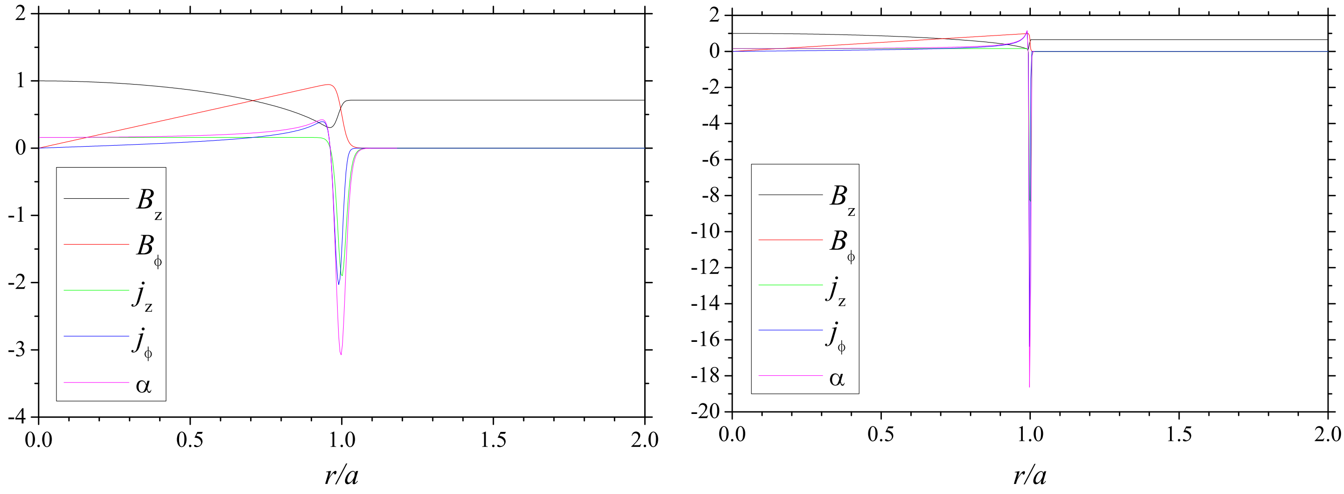

Fig. 2. Dependence of the equilibrium values of the magnetic field components B_z and B_φ, the electric current density j_z and j_φ, and the force-free parameter α on the relative cross-sectional radius r/a at $\kappa = 100$ (left panel) and $\kappa = 1000$ (right panel). The "gap" in the values of j_z, j_φ and α around $r/a = 1$ corresponds to the boundary region.

appearing both inside the flux rope and at the boundary. Indeed, from equation (3) for the components of the electric current density, we have

$$j_z = \frac{c}{4\pi}\frac{1}{r}\frac{\partial(rB_\varphi)}{\partial r}, \quad j_\varphi = -\frac{c}{4\pi}\frac{\partial B_z}{\partial r}. \tag{7}$$

For simplicity, taking the azimuthal magnetic field

$$B_\varphi = \frac{r/a}{1+(r/a)^\kappa}, \tag{8}$$

where $\kappa = \mathrm{const}$, from equations (2), (7), and (8) we find

$$\frac{B_z(r)}{B_z(0)} = \sqrt{2}\left(\frac{1}{2} - \frac{B_\varphi^2(r)}{2B_z^2(0)} - \frac{1}{B_z^2(0)}\int_0^r \frac{B_\varphi^2(r)}{r}dr\right)^{1/2}.$$

In this case the dimensionless parameter

$$\alpha = \frac{j_z}{B_z} = \frac{j_\varphi}{B_\varphi}. \tag{9}$$

The results of our numerical calculations of the magnetic field components B_z and B_φ, the electric current density j_z and j_φ, as well as the parameter α in arbitrary units at $\kappa = 100$ and $\kappa = 1000$ are shown in Fig. 2. As expected, no peculiarities for j and α are found, at least in the boundary region ($r = a$) implying a discontinuity, although the characteristic relative thickness of the boundary layer $\Delta r/a \to 0$, and j and α can reach arbitrarily large but finite values. From this trivial but illustrative example, it follows that the approximation of a sharp boundary to describe MHD equilibrium or the stability of a force-free magnetic flux rope is a fairly acceptable simplification. In the proposed model, unlike that in Solov'ev, Kirichek (2021), no special magnetic surfaces arise.

According to Solov'ev, Kirichek (2021), model (1) violates the continuity of the tangential component of the electric field $\mathbf{E}_\tau$ at the boundary. Meanwhile, in general case, the continuity of the component $\mathbf{E}^*_\tau$ for a boundary moving with velocity $\mathbf{v}$ can be expressed as in (Sommerfeld, 1949; Landau, Lifshitz, 1966; Miyamoto, 2000):

$$\langle(\mathbf{n}\times\mathbf{E}^*)\rangle = \mathbf{E}^*_{\tau e} - \mathbf{E}^*_{\tau i} = 0, \tag{10}$$

where $\mathbf{n}$ is the unit normal to the interface, and the electric field

$$\mathbf{E}^* = \mathbf{E} + \frac{1}{c}(\mathbf{v}\times\mathbf{B}). \tag{11}$$

Since within ideal MHD

$$\mathbf{E}^* = \mathbf{E} + \frac{1}{c}(\mathbf{v}\times\mathbf{B}) \approx 0, \tag{12}$$

equation (10) is automatically satisfied, and the question of the continuity of the tangential component of the electric field $\mathbf{E}^*_\tau$ within ideal plasma does not arise.

It is important to emphasize that the consideration of the plasma velocity $\mathbf{v}$ for force-free magnetic flux ropes has a profound physical significance, which was first noted by Alfvén (see, e.g., Alfvén, Fälthammar, 1967). Under the action of the electric field, the electron and ion drifts occur toward the rope axis at the speed

$$\mathbf{v} = \frac{c}{B^2}[\mathbf{E}\times\mathbf{B}].$$

On the other hand, taking into account equation (11), in a moving coordinate system, the electric field

$$\mathbf{E}^* = \mathbf{E} + \frac{1}{c}(\mathbf{v}\times\mathbf{B}) = \mathbf{E} + \frac{\mathbf{B}}{B}(\mathbf{E}\mathbf{B}) - \mathbf{E} = \mathbf{E}_\parallel.$$

It follows that only through the drift motion of the plasma, the formation of force-free magnetic structures in the solar corona becomes possible.

3 Discussion of results and conclusions

We have shown that the conclusion in Solov'ev, Kirichek (2021) on the possibility of forming special magnetic surfaces implying the presence of singularity for the electric current density and the parameter α inside equilibrium magnetic flux ropes, can hardly be considered convincing. In our view, this is due to an unsuccessful choice of the generating function $F(r)$, i.e., the solution to the MHD equilibrium equation (see also Solov'ev, 2022). Note that the approach used by the authors implies the flux rope radius $a \to \infty$ and, consequently, an infinitely large shielding region. Therefore, in this study, we have linked the characteristic flux rope boundary a to a thin layer where the magnetic field, electric current, and parameter α change quite abruptly.

As indicated by the obtained results, in the ideal MHD approximation, the continuity of the tangential component of the electric field E^* is automatically satisfied, as $E^* \approx 0$. Hence, it is obvious that when setting boundary conditions for the "plasma – plasma" discontinuity, this requirement is not usually taken into account since it "drops out". A detailed examination of the case associated with the consideration of finite electrical plasma conductivity implies the possibility of separating electric charges at the boundary surface, which is beyond the scope of this paper. However, as can be easily shown, in the case of high but finite electrical conductivity, the use of more general approaches is in good agreement with the equations of ideal MHD (Tsap, Shakhovskaya, 2000).

Thus, the models of laboratory pinches or the shielded magnetic flux rope can be used to describe MHD equilibrium and stability in the solar and stellar coronae, significantly simplifying model calculations (Tsap et al., 2020, 2022).

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